DETERMINING THE CURRENT DENSITY OF THE ELECTRODE SPOTS OF AN ELECTRIC ARC

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The energy balance and the Lambert-Bouguer law are used to obtain a mathematical relationship between the current density for the electrode spots of a carbon arc and the current. Theoretical calculations are confirmed by experiment.

The mean current density of the electrode spots in a freely burning arc has been measured by many research workers, but the answer to this question is still not completely clear. In particular, contrary to the widely held view, the current density depends very strongly on the arc current. For instance, Sisoyan [1] reported a cathode current density of 3600 A/cm^2 for an arc burning in air between copper electrodes with a current of 20 A. Cobine and Gallagher [2] reported a current density of $(50-220) \cdot 10^3$ A/cm^2 for the cathode spots of arcs burning between Al, Cu, and W electrodes with currents of about 140 A. Skeats and Schuck [3] found a mean cathode current density of 500–1000 A/cm^2 (maximum in center of column ~ 2500 A/cm^2) for an arc with a current of 12–14 kA burning in air between copper electrodes.

Similar results have been obtained for an arc burning in air between carbon electrodes. For instance, Zalesskii [4] evaluated the current density as ~125 A/cm^2 for the cathode spot of an arc with a current of 10 A; according to other data [1], it reaches 500 A/cm^2. Khrenov [6] found a current density of ~2800

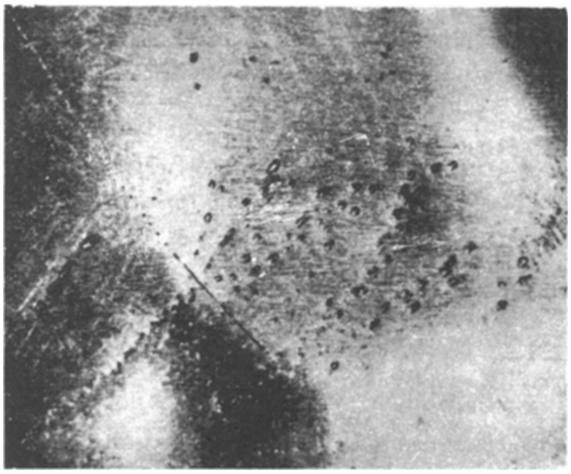

Fig. 1. Traces of arc combustion on brass foil intersecting arc column in anode region.

A/cm^2 for the cathode spot of arcs with a current of up to 600 A. It follows from [1] that for an arc with a current of about 40 kA the current density of the cathode spot is no more than 500 A/cm^2. Tikhodeev in an investigation of a welding arc (C^- and Fe^+ electrodes) with a current of 200–1000 A found that the current density in its column had a maximum value at a current of about 200 A and then decreased very rapidly with increase in current [7]. Finkelnburg and Maecker [8] cited Hillery's data. They showed that with increase in current, the cathode current density in carbon arcs increased from a few hundred amps per square centimeter to a maximum value of 5000 A/cm^2.

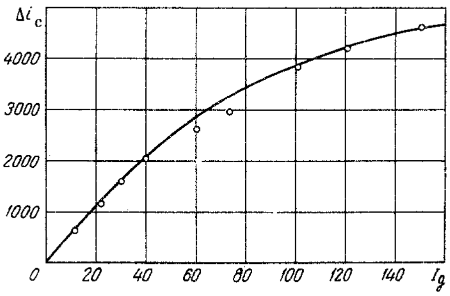

Fig. 2. Current density of cathode spot as a function of arc current between carbon electrodes in open air. The circles denote the experimentally derived points.

The cited data indicate that in arcs with a low current (without artificial cooling) the current density of the cathode spots is relatively low; at currents of 100–1000 A the current density of the spots increases to significant values and with further increase in current it again decreases very appreciably. We can conclude that such a peculiar, experimentally manifested variation for the current density of the electrode spots with arc current under the same combustion conditions is not accidential. We will attempt to explain this variation and propose a theoretical formula.

Sergeev in [5], making the generally accepted assumption that the energy from the electrode layers of an arc with carbon electrodes is removed mainly by radiation and using the energy-balance method ($U_e \Delta i_e F_{e.s} = \varepsilon\sigma T_a^4 F_{rad}$), derived a formula for the current density of the electrode spots

$$\Delta i_e = \frac{m \varepsilon T_a^4}{U_e} 5.67 \cdot 10^{-12} \text{ A/cm}^2, \tag{1}$$

where $m = F_{rad}/F_{e.s}$.

Measurement of arc temperatures is a special problem and we will not dwell on it in this paper. We merely note that with increase in current the temperature of an arc burning in open air, according to Brown's data (cited in [4]), increases insignificantly. In any case, with increase in current from 200 to 400 A the temperature increases by only ~3%. Its further increase with increase in current is even slower. Maecker [9] cites similar data for an arc burning between carbon electrodes in air. With increase in current from 200 to 500 amps, T_a at the cathode increases by approximately 4%. Hence, the change in arc temperature, according to formula (1), cannot account for such a peculiar and pronounced variation in the current density of the electrode spots with increase in arc current (particularly the reduction of density with increase in current).

Experimental and Calculated [from formula (6)] Cathode Current Densities for a Carbon Arc

Arc current, A	Value of m	Value of ε	Calculated cathode spot current density, A/cm^2	Experimental values, A/cm^2
11	0.130	1.000	615	633
100	0.713	0.995	3830	3800
500	1	0.970	5250	5000
1000	1	0.945	5100	5000
10000	1	0.610	3300	—
40000	1	0.131	700	~500

NOTE: For currents of 500, 1000, and 40 000 A, the current density is taken from the literature.

Many investigations have been devoted to determination of the electrode voltage drops due to liberation of heat energy in the spot region; this question has also been discussed in [5]. We note here that their value depends on the electrode material, the medium in which the arc burns, the pressure, and the ambient temperature. All these factors in the case of a carbon arc (also with copper electrodes) burning in open air are practically constant (due to increase in T_a some reduction in U_e can be expected). Hence, according to (1), the change in the values of the electrode voltage drops cannot account for the observed changes in the current density of the electrode spots with change in arc current. These changes are obviously due to the change in the values of m and ε as functions of the current. Let us find this relationship.

Henceforth, we will consider only the question of determining the current density of the electrode spots for arcs burning between carbon electrodes, and for which the formula is intended. In the case of arcs burning between metal electrodes, the energy balance used to construct formula (1) must include, in addition to radiation, the energy spent on melting and evaporating the electrodes. In particular, in the case of copper electrodes this expenditure of energy will be very much greater than for carbon electrodes, since the melting and boiling points of carbon electrodes are close to 4100° K, whereas for copper T_m = = 1356° K and T_b = 2903° K. This is probably why the current densities of the cathode spots in arcs burning between copper electrodes at low and medium currents are well in excess of the current densities on carbon electrodes. The heat loss from the layer near the electrode by convection and conduction plays an important role here.

We will determine m and ε by proceeding on the assumption that the electrode spots of carbon electrodes (as other electrodes) have a cellular structure and that the arc column (at least in the region adjacent to the electrode spots) accordingly has a fibrous structure. The cells comprising the complete spot are thus regarded as the beginnings or ends of the fibers—the current filaments which form the arc column and have high current density. The cellular structure of the cathode spot for mercury has been shown in the works of Froome [10] and Kesaev [11]. The authors of [3] believe that the structure of the cathode spot on a copper electrode is cellular. Many researchers have reported that electrode spots have a scaly structure [8].

Figure 1 shows the punctate structure of the anode spot of an arc burning between carbon electrodes in air. Air at a pressure of about $(9.8-19.6) \cdot 10^4$ N/m^2 was injected through an axial aperture in the upper electrode (cathode). The arc was supported on the flat surface of a graphite-coated anode, on which splitting of the arc was observed [5]. The punctate traces of the arc seen on the photograph (some of the spot points) were obtained on a sheet of brass foil which rapidly intersected the arc in the anode region (the same result was obtained in the center of the column). The set of points—traces of arc combustion on the foil—indicates the cellular structure of the anode spot and the filamentous structure of the arc column [12].

We assume, as is generally done, that the emissivity of the electrode spots of carbon arcs of low and medium current (without artificial cooling) is close to unity [8]. With increase in current, however, the intensity of emission of the electrode spots will be reduced owing to its absorption by the column in the region of the electrode spot. The reduction in emission intensity can be quantitatively accounted for by reduction of the emissivity ε.

Arcs with high current densities radiate like black bodies. Accordingly, the current filaments (elementary arcs) with the gas in their immediate vicinity can be regarded as black bodies.

If we consider the emission from the center of the electrode spot, it is quite obvious that under the same conditions the greater the arc current, the greater the number of current filaments in the region of the spot to absorb radiation.

We know that absorption of radiation by various media conforms to the Lambert–Bouguer law

$$q = q_0 \exp(-kx) \qquad (2)$$

and that absorption of radiation by substances dissolved in transparent solvents is proportional to the number of absorbing molecules in the path of the beam. In cases to which the considered one belongs, the absorption coefficient is proportional to the concentration of the solution, and the Lambert-Bouguer formula takes the form [13]

$$q = q_0 \exp(-zcx).$$

In our case the radiation is absorbed by the current filaments and hence, the total arc current will be proportional both to their concentration in the region of the column adjoining the spot (similar to the concentration of particles in the solution) and to the linear dimension x. In fact, if we consider the uniformly distributed emission of an elementary cell in the center of the electrode spot and divide the circular area of the spot into infinitely small sectors, we find that the absorption in each sector will be proportional to the concentration of current filaments in it, multiplied by the area of each sector, i.e., it will be proportional to the current flowing in this sector. If we consider all the radiation from the cell, the absorption will be proportional to the current of the entire arc. A similar argument can be put forward for each cell of the spot. Hence, the Lambert-Bouguer law in application to absorption of radiation of electrode spots by the arc column will take the form

$$q = q_0 \exp(-\chi I_a). \tag{3}$$

Since the reduction (in comparison with a black body) of the intensity of the flux per unit surface is characterized by the emissivity ε, we finally have

$$\frac{q}{q_0} = \varepsilon = \exp(-\chi I_a). \tag{4}$$

In formula (4) ε_0 is unity.

According to (4), at relatively low currents the emissivity of the electrodes spots of an arc with carbon electrodes can be regarded virtually as unity (there is little absorption of the radiation). At high currents (more than 1000 A) the emissivity will be much less than unity and hence, the current density for the electrodes spots, according to formula (1), will be low.

We will find the law of variation of the coefficient $m = F_{rad}/F_{e.s}$ with change in the arc current. At low currents $F_{rad}/F_{e.s} \ll 1$, since the sum of the emitting surfaces (round the spot cells) is much less than the surface of the luminous spot, which is assumed to be its current-conducting area $F_{e.s}$. Under these conditions the current filaments and the spot cells are dispersed and the total luminous spot is produced by the rapid motions (the "dance") of the current filaments in the spot region (in this case the spot temperature is low). With increase in arc current the current filaments in the column and in the cells of the spot become packed closer together by the heat and the luminous zones of the cells fuse. The spot in this case really represents the emitting area (in accordance with formula (1)], i.e., the whole spot area radiates uniformly from its cells: $F_{rad} = F_{e.s}$. Thus dispersion of the elementary current cells is the cause of low current density. With increase in current the cells are rapidly compacted, coefficient m approaches unity, and the current density increases to a maximum value.

The nature of the variation in m with arc current, as in the case of ε, can be determined by using the Lambert-Bouguer law and assuming that the spot temperature is constant, which corresponds approximately to the condition $F_{rad} = F_{e.s}$.

We argue as follows. At a certain current I_a we have $F_{rad} = F_{e.s}$, i.e., in this case m = 1 and the whole electrode spot uniformly emits the maximum energy from unit surface. According to formula (1), when $\varepsilon = 1$ the current density is a maximum: $\Delta i_{max} = T_a^4/U_e$. On reduction of current the reduction in intensity of emission per unit surface of the spot, owing to dispersal of its cells in the "dance" and the corresponding reduction of the current density for the electrode spots can also be regarded as due to absorption of radiation in the column adjacent to the electrode. From this aspect the relative change in the absorption intensity will be given by the ratio $F_{rad}/F_{e.s}$, since F_{rad} is only the emitting part of the spot, whereas $F_{e.s}$ is its total area, including the "dark" regions covered by the "dance" and which also characterize the absorption. The absorption flux is $\Delta q = q_0 - q$. The flux q, attenuated by absorption, is given by expression (3) and, hence, the absorption flux can be determined from the formula

$$\Delta q = q_0 - q_0 \exp(-\chi I_a).$$

The relative reduction of the maximum emission flux is given by the expression

$$\frac{\Delta q}{q_0} = 1 - \exp(-\chi I_a),$$

and there will be a corresponding change in the maximum current density in the electrode spots. Thus the coefficient

$$m = 1 - \exp(-\chi' I_a). \tag{5}$$

In (5) the coefficient χ' is not equal to the χ in formula (4).

Combining (1), (4), and (5) we obtain a more general expression for the current density of the electrode spots in arcs with carbon electrodes:

$$\Delta i_e = \frac{T_a^4}{U_e} \exp(-\chi I_a) \times$$

$$\times [1 - \exp(-\chi' I_a)] \cdot 5.67 \cdot 10^{-12}\ \mathrm{A/cm^2}. \tag{6}$$

If we take the first derivative $d\Delta i_e/dI_a$ and equate it to zero, we find at what arc current I_a the maximum electrode current density occurs. By simple transformations we obtain

$$I_{a(\Delta i_e = \max)} = \frac{\log\left(\frac{\chi'}{\chi} + 1\right)}{0.434\chi'}. \tag{7}$$

In formula (6) the current density of the electrode spots depends on the electrode material, the medium in which the arc burns, as well as on the pressure and the ambient temperature, since T_a and U_e depend on these factors, and also on the arc current. Thus, though the formula is comparatively simple, it takes a great variety of factors into account. In deriving it we had in mind only freely burning arcs without forced cooling, but it can be used in principle for intensively cooled arcs.

Finally, even in an arc burning in air between carbon electrodes without forced cooling (or heating) T_a and U_e can vary over some range (due to change in ambient temperature, design of electrodes and apparatus as a whole, etc.) and, hence, even at the same arc currents the electrode spot current densities may vary a little.

By photographing the electrode spots we experimentally determined the current density in cathode spots of a vertically burning arc between carbon (graphite-coated) electrodes in air in relation to the arc current, i.e., we obtained the function $\Delta i_c = f(I_a)$. Photographs of the lower flat electrode (diameter 150 mm) were taken through an orange + yellow filter at an exposure of 10^{-3} sec. The upper electrode of diameter 30 mm had a sharp tip. The arc length was 2–3 cm. The arc currents varied from 11–150 A. The results of the experiment are shown in Fig. 2 (Engineer R. Gabbasova participated in doing the experiment). The relationship $\Delta i_c = f(I_a)$ in the range of the experiment is approximated to within 5% by the exponential relationship

$$\Delta i_c = 5400\,[1 - \exp(-1.25 \cdot 10^{-2} I_a)], \tag{8}$$

i.e., an exponential relationship of the same kind as formula (5).

Expression (8) shows that the proportionality factor χ' in formula (5) is equal to $1.25 \cdot 10^{-2}$ A^{-1} in the given conditions. As Fig. 2 (curve) shows, the current density of the cathode spot tends to a maximum at 5400 A/cm^2.

If we use Hillery's data, cited in [8], which indicate that the maximum cathode current density occurs at a current of ~400 A, from formula (7) we find that for this case the proportionality factor χ in formulas (4) and (6) will be approximately $5 \cdot 10^{-5}$ A^{-1}. This is very close in value to the optical absorption coefficient of air at atmospheric pressure, ~10^{-5} cm^{-1}.

It is important to find out how closely the experimentally observed maximum of cathode current density (5400 A/cm^2) corresponds to formula (6), in which it is expressed in the form

$$\Delta i_c = \frac{T_a^4}{U_e} 5.67 \cdot 10^{-12}\ A/cm^2.$$

We take the cathode voltage drop (due to heat release) U_e = 10.0 V [5], which is close to the ionization potential of nitric oxide intensively formed in an arc. For a current density of 5400 A/cm^2 the mean temperature in the cathode region space must be ~10 000° K. Maecker [9] obtained values close to this in an investigation of a carbon arc burning in air. Thus, the values contained in the formula for the maximum current density are very close to the experimentally observed values.

Table 1 gives values calculated from formula (6) for the current density of the cathode spots on an arc burning between carbon electrodes in open air (without artificial cooling) in relation to the arc current.

As the table shows, the cathode-spot current density depends greatly on the current. For anode spots the obtained relationship also holds, but the values of the proportionality factors χ in formulas (4), (5), and (6) will be different.

According to (6), in arcs of very high current we have low electrode-spot current densities and dispersal of energy. This obviously explains why a high-quality product can be obtained in electric-arc furnaces only if the arc current does not exceed a particular value [14].

In conclusion we note that the current density of the electrode spots on a carbon arc burning in open air is a function of its current. The function has a maximum, the value of which depends on the electrode voltage drop and the temperature of the electrode region. The position of the current density maximum on the current axis is determined by the coefficients χ and χ'.

NOTATION

Δi_e is the current density in the electrode spot, A/cm^2; Δi_c is the current density in the cathode spot, A/cm^2; T_a is the mean temperature in the electrode region, °K; U_e is the electrode voltage drops due to heat release, V; I_a is the arc current, A; ε is the emissivity of electrode spot; $m = F_{rad}/F_{e.s}$; F_{rad} is the emitting area of spot; $F_{e.s}$ is the measured electrode spot area, assumed to be current-conducting; q_0 is the radiation flux density before absorption; q is the same after absorption; k is the absorption coefficient; z is a proportionality factor; c is the concentration of solution; x is a linear dimension (thickness); e is the base of natural logarithms; χ, χ' are the proportionality factors (radiation absorption coefficients), A^{-1}.

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